Space Physics Research Laboratory Memorandum

21 May 2002

To: TIDI File
From: David Gell
Subject: Coordinate Frames and Viewing Directions

0. Revision History

0.1 Revision B, 21 May 2002

Adds the attitude reference frame as described in reference 3.

0.2 Revision A, 17 November 1999

Corrects the sense of the rotation between the telescope coordinate frame and the spacecraft frame. Adds equation numbers. Corrects the signs of the sine terms in equation 9.

1. References

1 Wertz, James R.(editor), *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Coompany, Inc. 1978

2 J.O. Cappellari, C.E. Velez, and A.J. Fuchs, "Mathematical Theory of the Goddard Trajectory Determination System", Goddard Space Flight Center, X-582-76-77, April 1976

3 Dellinger, Wayne, Private Communication, 17 May 2002

2. Introduction

This memo defines coordinate frames and pointing directions for use in TIDI data processing and modeling. The coordinate frames described are the spacecraft (sc) frame, telescope (t) frames, the attitude reference (ar) frame, the local horizontal-local vertical (lhlv) frame, and the Earth centered inertial (eci) frame. Besides describing these reference frames, the transformations between them will be specified.

Coordinate frames are often defined by specifying an origin for the coordinates, a reference plane, and a principal direction. The principal direction lies within the reference plane. Transformations between coordinate systems are specified by transformation matrices.

Within this memo, the notation

$$\mathbf{T}_{\frac{x}{y}}$$

refers to a rotation matrix which transforms a vector from the y to the x coordinate frame. Vectors are denoted by a bold face symbol, with a superscript denoting the coordinate frame in which the coordinates of the vector are expressed:

$$\mathbf{v}^{(eci)}$$

3. Coordinate Frame Definitions

3.1 Earth Centered Inertial Frame.

The Earth centered inertial (eci) frame originates at the Earth's center. The reference plane is the Equator, and the principal direction is the Vernal Equinox. The Vernal Equinox is the intersection of the earth's equator and the earth's orbital plane, with the positive direction being from the earth to sun at the spring (northern hemisphere) equinox.

3.2 Local Horizontal - Local Vertical Frame

The local horizontal - local vertical (lhlv) frame originates at the spacecraft center of mass. The reference plane is the local horizontal, that is the plane normal to the vector from the Earth's center to the spacecraft center of mass. The principal direction $\mathbf{x}^{(lhlv)}$ is the projection of the velocity vector onto the reference plane. The $\mathbf{z}^{(lhlv)}$ axis points towards the Earth's center, the $\mathbf{y}^{(lhlv)}$ axis completes the right hand coordinate system and points in the direction of the negative orbital angular moment.

3.3 The Attitude Reference Frame

The attitude reference (ar) frame originates at the spacecraft center of mass. The reference plane is the local horizontal, that is the plane normal to the vector from the Earth's center to the spacecraft center of mass. In forward flight, the principal direction $\mathbf{x}^{(ar)}$ is the projection of the velocity vector onto the reference plane. In backward flight, the principal direction $\mathbf{x}^{(ar)}$ is such that the projection of the velocity vector onto the reference plane is $-\mathbf{x}^{(ar)}$. The $\mathbf{z}^{(ar)}$ axis points towards the Earth's center, the $\mathbf{y}^{(ar)}$ axis completes the right hand coordinate system and points in the direction of the negative orbital angular moment.

In forward flight the attitude reference frame and the local horizontal – local vertical frames are identical. In backwards flight, the attitude reference frame is rotated 180° about the $\mathbf{z}^{(lhlv)}$ axis from the local horizontal – local vertical frame.

3.4 The Spacecraft Frame

The spacecraft frame is fixed to the spacecraft. Its origin is the spacecraft center of mass. The reference plane is the plane containing the roll and yaw axes. The principal direction is along the roll axis towards the front of the spacecraft. These axes are specified as part of the mechanical design of the spacecraft. The axes are denoted by $\mathbf{x}^{(sc)}$, $\mathbf{y}^{(sc)}$ and $\mathbf{z}^{(sc)}$ are the roll, pitch and yaw axes, respectively.

The TIMED attitude control system attempts to maintain the spacecraft frame and the attitude reference frames coincident.

3.5 Telescope Frames

There is one telescope frame for each of the four TIDI telescopes. Each telescope frame originates at intersection of the telescope's gimbal axis and its optical axis is defined with its reference plane parallel to the spacecraft's. The principal direction is the nominal azimuth of the telescope projected on the reference frame. The $\mathbf{z}^{(t)}$ axis is parallel to the $\mathbf{z}^{(sc)}$ axis. Table 1 lists the identifies the nominal azimuths α_0 for each of the telescopes

Table 1, Telescope Nominal Azimuths

telescope	package ID	nominal azimuth
1	A300	45
2	A301	135
3	A302	225
4	A303	315

Within each telescope frame, the viewing direction is given by

$$\mathbf{l}^{(t)} = \begin{pmatrix} \cos\alpha'\cos\varepsilon \\ \sin\alpha'\cos\varepsilon \\ \sin\varepsilon \end{pmatrix} \quad (1)$$

where α' is the azimuth offset from the nominal, and ε is the elevation angle measured from the reference plane towards the $z^{(t)}$ axis.

4. Coordinate Transformations

4.1 From Local Horizontal Local Vertical to ECI

The coordinates of a vector $\overline{\mathbf{p}}^{(b)}$ in the local horizontal local vertical frame can be expressed in the ECI frame by:

$$\overline{\mathbf{p}}^{(eci)} = \mathbf{T}_{\frac{eci}{lhlv}} \overline{\mathbf{p}}^{(lhlv)} \tag{2}$$

Using the fact the columns of a transformation matrix are the unit vectors of the untransformed frame in the transformed frame, the rotation matrix is:

$$\mathbf{T}_{\frac{eci}{lhlv}} = \left[\left(\hat{\mathbf{r}}^{(eci)} \times \hat{\mathbf{v}}^{(eci)} \right) \times \hat{\mathbf{r}}^{(eci)} \quad \middle| -\left(\hat{\mathbf{r}}^{(eci)} \times \hat{\mathbf{v}}^{(eci)} \right) \middle| \quad -\hat{\mathbf{r}}^{(eci)} \right] \tag{3}$$

where $\hat{\mathbf{r}}^{(\mathbf{eci})}$ and $\hat{\mathbf{v}}^{(\mathbf{eci})}$ are unit vectors in the direction of the spacecraft position and velocity expressed in the ECI coordinate frame. Setting

$$\hat{\mathbf{r}}^{(eci)} = \begin{pmatrix} \mathrm{r}_1 \\ \mathrm{r}_2 \\ \mathrm{r}_3 \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{v}}^{(\mathbf{eci})} = \begin{pmatrix} \mathrm{v}_1 \\ \mathrm{v}_2 \\ \mathrm{v}_3 \end{pmatrix} \tag{4}$$

the transformation matrix in terms of the components of the velocity and position is

$$\mathbf{T}_{\frac{eci}{lhlv}} = \begin{pmatrix} v_1\left(r_2^2 + v_3^2\right) - r_1\left(r_2 v_2 + r_3 v_3\right) & r_3 v_2 - r_2 v_3 & -r_1 \\ v_2\left(r_1^2 + v_3^2\right) - r_2\left(r_1 v_1 + r_3 v_3\right) & r_1 v_3 - r_3 v_1 & -r_2 \\ v_3\left(r_1^2 + v_2^2\right) - r_3\left(r_1 v_1 + r_2 v_2\right) & r_2 v_1 - r_1 v_3 & -r_3 \end{pmatrix} \tag{5}$$

4.2 From Attitude Reference To Local Horizontal – Local Vertical

The coordinates of a vector $\overline{\mathbf{p}}^{(\mathrm{ar})}$ in the attitude reference frame can be expressed in the local horizontal local vertical frame by:

$$\overline{\mathbf{p}}^{(lhlv)} = \mathbf{T}_{\frac{lhlv}{ar}} \overline{\mathbf{p}}^{(ar)} \tag{6}$$

When the spacecraft is in forward flight, the rotation matrix is the identity matrix. When in backward flight, the rotation matrix is

$$\mathbf{T}_{\frac{lhlv}{ar}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

4.3 From Spacecraft Frame To Attitude Reference Frame

The coordinates of a vector $\overline{\mathbf{p}}^{(sc)}$ in the spacecraft frame can be expressed in the attitude reference frame by:

$$\overline{\mathbf{p}}^{(ar)} = \mathbf{T}_{\frac{ar}{sc}} \overline{\mathbf{p}}^{(sc)} \tag{8}$$

The transformation between the spacecraft frame and the attitude reference frame $\mathbf{T}_{\frac{ar}{sc}}$ is a function of the spacecraft attitude. In general the rotation matrix is

$$\mathbf{T}_{\frac{ar}{sc}} = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\theta & \sin\theta \\ \cos\psi\sin\theta\sin\phi + \sin\psi\cos\phi & -\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & -\cos\theta\sin\phi \\ -\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi + \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix} \tag{9}$$

where ϕ, θ, and ψ are the rotations about the $\mathbf{x}^{(sc)}$, $\mathbf{y}^{(sc)}$, and $\mathbf{z}^{(sc)}$ axes (roll, pitch, and yaw). For small attitude errors, the transformation is

$$\overline{\mathbf{p}}^{(ar)} = \begin{bmatrix} 1 & -\Delta\psi & \Delta\phi \\ \Delta\psi & 1 & -\Delta\theta \\ -\Delta\phi & \Delta\theta & 1 \end{bmatrix} \overline{\mathbf{p}}^{(sc)} \tag{10}$$

If there are no attitude errors, the transformation matrix is the identity matrix.

4.4 Telescope to Spacecraft

The transformation from the telescope frame to the spacecraft frame is a negative rotation about the telescope z axis through the nominal azimuth α_0, as given in Table 1. The rotation matrix is

$$\mathbf{T}_{\frac{sc}{t}} = \begin{pmatrix} \cos\alpha_0 & -\sin\alpha_0 & 0 \\ \sin\alpha_0 & \cos\alpha_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

5. Line of Sight Direction

With the transformation matrices defined in section 4, the viewing direction for a particular telescope and elevation angle can be expressed in any of the coordinate frames.

The viewing vector in the telescope frame is given by

$$\mathbf{l}^{(t)} = \begin{pmatrix} \cos\alpha' \cos\varepsilon \\ \sin\alpha' \cos\varepsilon \\ \sin\varepsilon \end{pmatrix} \tag{12}$$

where α' is the azimuth offset from the nominal, and ε is the elevation angle measured from the reference plane towards the $z^{(t)}$ axis. The elevation angle increases as the tangent altitude decreases.

The viewing direction in any of the other reference frames may be obtained by successive rotations. The following table lists the viewing direction in each of the coordinate frames.

Table 2, Viewing Direction

frame	viewing direction
telescope	$\mathbf{l}^{(t)}$
spacecraft	$\mathbf{l}^{(sc)} = \mathbf{T}_{\frac{sc}{t}}\mathbf{l}^{(t)}$
attitude reference	$\mathbf{l}^{(ar)} = \mathbf{T}_{\frac{ar}{sc}}\mathbf{T}_{\frac{sc}{t}}\mathbf{l}^{(t)}$
local horizontal - local vertical	$\mathbf{l}^{(lhlv)} = \mathbf{T}_{\frac{lhlv}{ar}}\mathbf{T}_{\frac{ar}{sc}}\mathbf{T}_{\frac{sc}{t}}\mathbf{l}^{(t)}$
eci	$\mathbf{l}^{(eci)} = \mathbf{T}_{\frac{eci}{lhlv}}\mathbf{T}_{\frac{lhlv}{ar}}\mathbf{T}_{\frac{ar}{sc}}\mathbf{T}_{\frac{sc}{t}}\mathbf{l}^{(t)}$