Memo to: TIDI
From: W. R. Skinner
Date: September 5, 2003
Subject: Revised VECTOR wind algorithm
TIDI memo: 055-4263b

I. Introduction

This memo presents a simple algorithm that can be used to determine vector winds from line of sight velocities. This algorithm supercedes the one described in TIDI document 055-4079.

II. Zonal and Meridional Wind from line of sight winds

The line of sight (v_{los}) wind is related to the zonal (u) and meridional (v) wind by

$$v_{los} = -u \sin \Phi - v \cos \Phi$$

where Φ is the look direction at the tangent point from north. Four measurements, two from forward views and two from backward views that bracket the desired location are used in the analysis as shown in figure 1.

Between measurements, the zonal and meridional winds are assumed by vary linearly:

$$u(\theta_t) = u_0 + \alpha\theta_t$$

$$v(\theta_t) = v_0 + \beta\theta_t$$

where θ_t is the tangent point track angle and u_0, v_0, α, β, are coefficients that describe the

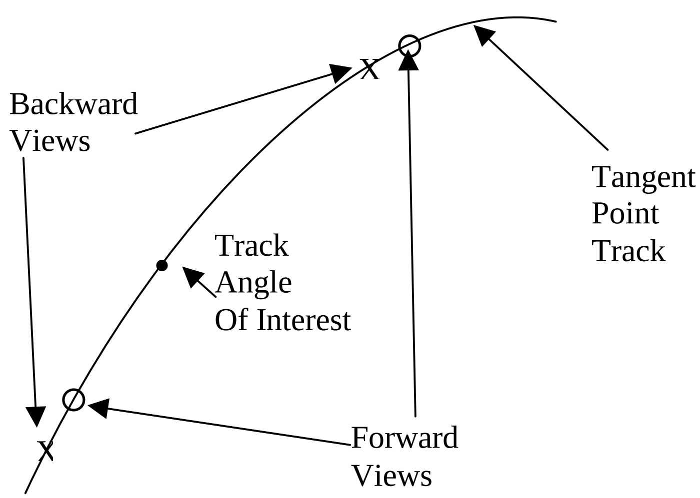

Figure 1. Illustration of the observational geometry

linear trend. Once these are determined, the desired wind components can be calculated. The four measurements can be expressed:

$$v_{los,1} = -u_1 \sin\Phi_1 - v_1 \cos\Phi_1$$

$$v_{los,2} = -u_2 \sin\Phi_2 - v_2 \cos\Phi_2$$

$$v_{los,3} = -u_3 \sin\Phi_3 - v_3 \cos\Phi_3$$

$$v_{los,4} = -u_4 \sin\Phi_4 - v_4 \cos\Phi_4 .$$

Substituting the linear equations for u and v give

$$v_{los,1} = -(u_0 + \alpha\theta_{t,1})\sin\Phi_1 - (v_0 + \beta\theta_{t,1})\cos\Phi_1$$

$$v_{los,2} = -(u_0 + \alpha\theta_{t,2})\sin\Phi_2 - (v_0 + \beta\theta_{t,2})\cos\Phi_2$$

$$v_{los,3} = -(u_0 + \alpha\theta_{t,3})\sin\Phi_3 - (v_0 + \beta\theta_{t,3})\cos\Phi_3$$

$$v_{los,4} = -(u_0 + \alpha\theta_{t,4})\sin\Phi_4 - (v_0 + \beta\theta_{t,4})\cos\Phi_4 .$$

This can be written in matrix form

$$\begin{bmatrix} v_{los,1} \\ v_{los,2} \\ v_{los,3} \\ v_{los,4} \end{bmatrix} = \begin{bmatrix} -\sin\Phi_1 & -\cos\Phi_1 & -\theta_{t,1}\sin\Phi_1 & -\theta_{t,1}\cos\Phi_1 \\ -\sin\Phi_2 & -\cos\Phi_2 & -\theta_{t,2}\sin\Phi_2 & -\theta_{t,2}\cos\Phi_2 \\ -\sin\Phi_3 & -\cos\Phi_3 & -\theta_{t,3}\sin\Phi_3 & -\theta_{t,3}\cos\Phi_3 \\ -\sin\Phi_4 & -\cos\Phi_4 & -\theta_{t,4}\sin\Phi_4 & -\theta_{t,4}\cos\Phi_4 \end{bmatrix} \begin{bmatrix} u_0 \\ v_0 \\ \alpha \\ \beta \end{bmatrix} .$$

Define

$$\mathbf{K} = \begin{bmatrix} -\sin\Phi_1 & -\cos\Phi_1 & -\theta_{t,1}\sin\Phi_1 & -\theta_{t,1}\cos\Phi_1 \\ -\sin\Phi_2 & -\cos\Phi_2 & -\theta_{t,2}\sin\Phi_2 & -\theta_{t,2}\cos\Phi_2 \\ -\sin\Phi_3 & -\cos\Phi_3 & -\theta_{t,3}\sin\Phi_3 & -\theta_{t,3}\cos\Phi_3 \\ -\sin\Phi_4 & -\cos\Phi_4 & -\theta_{t,4}\sin\Phi_4 & -\theta_{t,4}\cos\Phi_4 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_{los,1} \\ v_{los,2} \\ v_{los,3} \\ v_{los,4} \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} u_0 \\ v_0 \\ \alpha \\ \beta \end{bmatrix}$$

giving the matrix equations

$$\mathbf{v} = \mathbf{kx}$$

$$\mathbf{k}^{\mathbf{T}}\mathbf{v} = \mathbf{k}^{\mathbf{T}}\mathbf{k}\mathbf{x}$$

$$\mathbf{x} = \left(\mathbf{k}^{\mathbf{T}}\mathbf{k}\right)^{-1}\mathbf{k}^{\mathbf{T}}\mathbf{v}$$

and let

$$\mathbf{K_i} = \left(\mathbf{k}^{\mathbf{T}}\mathbf{k}\right)^{-1}\mathbf{k}^{\mathbf{T}}$$

The four coefficients are then

$$u_0 = \sum_{j=1}^{4} K_i(1,j)v_{los,j}$$

$$v_0 = \sum_{j=1}^{4} K_i(2,j)v_{los,j}$$

$$\alpha = \sum_{j=1}^{4} K_i(3,j)v_{los,j}$$

$$\beta = \sum_{j=1}^{4} K_i(4,j)v_{los,j}\ .$$

The interpolated values of the wind are then

$$u(\theta_t) = \sum_{j=1}^{4}\left(K_i(1,j) + \theta_t K_i(3,j)\right)v_{los,j}$$

$$v(\theta_t) = \sum_{j=1}^{4}\left(K_i(2,j) + \theta_t K_i(4,j)\right)v_{los,j}\ .$$

The uncertainties in u and v are readily determined assuming the errors in the line of sight speeds are independent:

$$\sigma^2_{u(\theta_t)} = \sum_{j=1}^{4}\left(K_i(1,j) + \theta_t K_i(3,j)\right)^2 \sigma^2_{v_{los,j}}$$

$$\sigma^2_{v(\theta_t)} = \sum_{j=1}^{4}\left(K_i(2,j) + \theta_t K_i(4,j)\right)^2 \sigma^2_{v_{los,j}}\ .$$

III. Interpolated scalar quantities

To find a scalar quantity at a desired track angle, the scalar variable at the same 4 reference positions are used. The 2 forward views that bracket the desired track angle are interpolated, the 2 backward views are interpolated, and finally the interpolated values are averaged. Assuming points 1 and 2 are either the forward or backward view and w is the scalar quantity of interest, then the first interpolated value is

$$w_1(\theta_t) = \frac{w(\theta_{t,1})(\theta_t - \theta_{t,2}) - w(\theta_{t,2})(\theta_t - \theta_{t,1})}{\theta_{t,1} - \theta_{t,2}}\ .$$

The second interpolated value is

$$w_2(\theta_t) = \frac{w(\theta_{t,3})(\theta_t - \theta_{t,4}) - w(\theta_{t,4})(\theta_t - \theta_{t,3})}{\theta_{t,3} - \theta_{t,4}}.$$

This gives

$$\begin{aligned} w(\theta_t) &= \frac{1}{2}\left(w_1(\theta_t) + w_2(\theta_t)\right) \\ &= \frac{w(\theta_{t,3})(\theta_t - \theta_{t,4}) - w(\theta_{t,4})(\theta_t - \theta_{t,3})}{2\left(\theta_{t,3} - \theta_{t,4}\right)} + \frac{w(\theta_{t,1})(\theta_t - \theta_{t,2}) - w(\theta_{t,2})(\theta_t - \theta_{t,1})}{2\left(\theta_{t,1} - \theta_{t,2}\right)}. \end{aligned}$$

Assuming each w is independent, the variance in the interpolated value is

$$\sigma^2_{w_{\theta_t}} = \frac{\sigma^2_{w_{\theta_{t,3}}}(\theta_t - \theta_{t,4})^2 - \sigma^2_{w_{\theta_{t,4}}}(\theta_t - \theta_{t,3})^2}{4\left(\theta_{t,3} - \theta_{t,4}\right)^2} + \frac{\sigma^2_{w_{\theta_{t,1}}}(\theta_t - \theta_{t,2})^2 -)\sigma^2_{w_{\theta,2t}}(\theta_t - \theta_{t,1})^2}{4\left(\theta_{t,1} - \theta_{t,2}\right)^2}.$$