University of Michigan
Space Physics Research Laboratory

Oblateness and Attitude Compensation	CAGE No.	0TK63
	Drawing No.	055-3662D
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REVISION RECORD

Rev	Description	Date	Approval
D	• Attitude compensation does not depend on direction of flight • Add corrected derivation of latitude and longitude intervals as an appendix, with a note in section 6.1 that the intervals used in the flight algorithm are an approximation, valid near the equator	1-Nov-2002	
C	• Corrected IDL routine that evaluates eqn 13, reproduced tables and figures in the appendix • Revised tables 5, 6, 7, & 8, to use true conversion from angle to encoder digital number	21 Apr 1999	
B	• Editorial corrections • Corrected equations 2, 8, & 22 • Corrected sign in equation 15	19 Apr 1999	
A1	• Corrected equation for C (eqn 2) • Corrected equation cross reference in section 7.2	4 Feb 1999	
A	• Add algorithm for orbital eccentricity compensation • Corrected signs in equations 4 and 4 • Correct signs in table 4	4 Jan 1999	
	Initial Release	8 Sep 1998	

APPROVAL RECORD

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List of Symbols

symbol	explanation
$\hat{\ell}$	unit vector in the direction of the telescope line of sight
$\mathbf{T}_{\frac{lhlv}{sc}}$	rotation matrix which transforms a vector in the spacecraft frame to the local horizontal local vertical frame
$\mathbf{T}_{\frac{sc}{t}}$	rotation matrix which transforms a vector in the telescope frame to the spacecraft frame
□	spacecraft pitch angle
□	spacecraft roll angle
□	spacecraft yaw angle
A	geographic azimuth of the line of sight
$□_0$	telescope azimuth angle, measured in the spacecraft coordinate frame
□	viewing angle, the angle between a telescope line of sight and the local horizontal
C	term in the definition of d
□	geodetic latitude
□□	co-latitude □= 90 □ □
d	distance from the center of the earth to a point on the reference ellipsoid, in multiples of R_e.
□□	difference in latitude between the tangent point and the satellite
□□	generic attitude error compensation, eqn 28
$□□_□$	compensation in telescope gimbal angle for roll attitude error
$□□_□$	compensation in telescope gimbal angle for pitch attitude error
□□	difference in longitude between the tangent point and the satellite
$□_□$	Orbital eccentricity correction factor in equation 10.
□	telescope gimbal angle, measured positive downward from the spacecraft x-y plane
f	flattening of the earth, 1/298.257223563
□	spacecraft heading angle, measured from east.
i	orbital inclination
K	Constant, value +1 or -1 in attitude error compensations
□	longitude
L	distance from the spacecraft to the tangent point along the line of sight
□	generic attitude error, eqn 28.
R(□)	radius of the earth as a function of latitude
R_e	equatorial radius of the earth, 6378.137 km
R_S	distance from the spacecraft to the center of the earth
R_{S_o}	nominal orbital radius, 7003 km
R_T	distance from the tangent point to the center of the earth
S	term in the definition of d
t	orbital track angle, sum of the true anomaly and the argument of perigee.
v	true anomaly
□	argument of perigee
□	longitude of the ascending node
Z_t	tangent point altitude, km

1. References

1) Gell, D. A., "Coordinate Frames and Viewing Directions", SPRL File 055-3543, 29 Jan 1998

2) DeMajistre, R., "TIMED Position and Attitude Geometry Description" APL Document SRS-98-157, 25 August 1998

3) Larson, W.J and Wertz, J.R. (editors), *Space Mission Analysis and Design, Second Edition*, Kluwer Academic Publishers, 1992

2. Introduction

The TIDI instrument scans the earth's limb to make atmospheric measurements. The measurements are to be at specified altitudes above the surface of the earth, but the mechanism is commanded in angle relative to the spacecraft coordinate frame (Ref. 1). The angle required to view a particular altitude can be readily computed assuming that the spacecraft orbit is circular, that the earth is spherical and that the spacecraft attitude is perfect. These assumptions are not met for TIDI. The spacecraft orbital radius can vary by up to 25 km. The radius of the earth varies from 6378 km at the equator to 6356 km at the poles which, uncorrected, can cause up to a 20 km error in tangent point altitude. In addition the spacecraft attitude is controlled with a precision of 0.5 degrees which, uncorrected, can cause altitude errors of 25 km.

The approach taken on TIDI is to specify the elevation angles assuming a circular orbit about a constant radius earth in the instrument control programs and correct for these effects in the flight software. Three compensation algorithms are required. One specifies the correction for spacecraft orbital eccentricity as a function of altitude excursion. The next specifies, for each telescope, the pointing correction which compensate for the earth's oblateness as a function of the spacecraft latitude The last specifies the corrections to the pointing to compensate for the actual spacecraft attitude.

This memo develops the theory behind the compensation tables. The compensation for orbital eccentricity is developed in section 5, *Orbital Eccentricity Correction* The compensation for oblateness is developed in section 6, *Oblateness Correction*. The compensation for actual attitude is in section 7, *Attitude Errors*.

3. The Figure of the Earth

The radius of the earth as a function of geodetic latitude ϕ is (ref. 3)

$$R(\phi) = R_e d \tag{1}$$

where d is a function of the flattening factor and geodetic latitude as follows:

$$\begin{aligned} C &= \left[\cos^2\phi + (1-f)^2 \sin^2\phi\right]^{-\frac{1}{2}} \\ S &= (1-f)^2 C \\ d^2 &= \tfrac{1}{2}\left(S^2 + C^2\right) + \tfrac{1}{2}\left(C^2 - S^2\right)\cos 2\phi \end{aligned} \tag{2}$$

The radius as a function of geodetic latitude is tabulated in Table 1 for the range of latitudes that the TIMED spacecraft will cover.

Table 1, Earth Radius

geodetic latitude deg	radius km	geodetic latitude deg	radius km
-74.5	6358.29	1.0	6378.13
-74.0	6358.39	6.0	6377.91
-69.0	6359.52	11.0	6377.37
-64.0	6360.89	16.0	6376.52
-59.0	6362.46	21.0	6375.41
-54.0	6364.18	26.0	6374.06
-49.0	6366.00	31.0	6372.50
-44.0	6367.86	36.0	6370.79
-39.0	6369.71	41.0	6368.98
-34.0	6371.49	46.0	6367.12
-29.0	6373.14	51.0	6365.26
-24.0	6374.62	56.0	6363.48
-19.0	6375.89	61.0	6361.81
-14.0	6376.90	66.0	6360.32
-9.0	6377.62	71.0	6359.04
-4.0	6378.03	74.5	6358.29

4. Viewing Geometry

The basic viewing geometry is shown in Figure 1. In that figure point C is the center of the earth, S is the spacecraft location and T is the tangent point. The tangent point altitude is Z_t. The latitude of the tangent point is $\square_t$, the latitude of the spacecraft is $\square_s$, the viewing direction measured from the local horizontal at the spacecraft is $\square$. The equatorial radius of the earth is R_e and the radius at the latitude of the tangent point is $R(\square_t)$.

The instrument viewing direction is denoted by the unit vector $\hat{\ell}$, which may be expressed in terms of the gimbal elevation angle $\square$, the telescope azimuth $\square_0$, and the spacecraft roll, pitch and yaw angles, $\square$, $\square$, and $\square$ (ref. 1).

$$\hat{\ell}^{(lhlv)} = \mathbf{T}_{\frac{lhlv}{sc}} \mathbf{T}_{\frac{sc}{t}} \begin{bmatrix} \cos\square \\ 0 \\ \sin\square \end{bmatrix} \tag{3}$$

$$\hat{\ell}^{(lhlv)} = \begin{bmatrix} 1 & \square\square & \square \\ \square & 1 & \square\square \\ \square\square & \square & 1 \end{bmatrix} \begin{bmatrix} \cos\square_0 & \square\sin\square_0 & 0 \\ \sin\square_0 & \cos\square_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\square \\ 0 \\ \sin\square \end{bmatrix} \tag{4}$$

The viewing angle β is obtained from dot product of the viewing direction $\hat{\ell}$ and a unit vector in the direction of the z axis in the local horizontal local vertical frame:

$$\begin{aligned} \sin\beta &= \hat{\mathbf{z}}^{(lhlv)} \bullet \hat{\ell}^{(lhlv)} \\ &= \sin\square\square\square\cos\square_0\cos\square + \square\sin\square_0\cos\square \end{aligned} \tag{5}$$

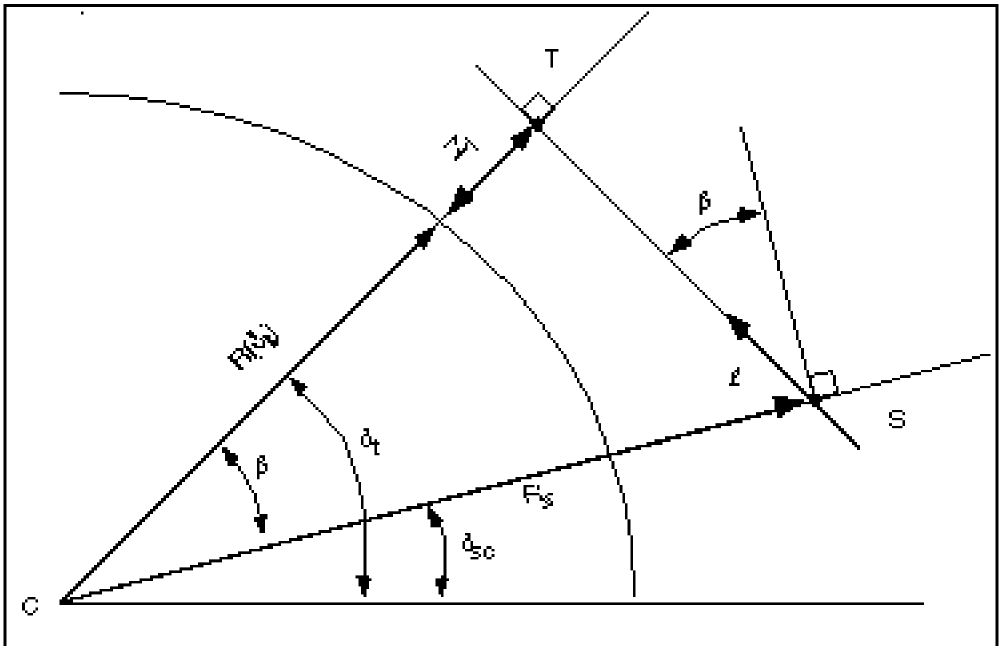

Figure 1, Limb Viewing Geometry

5. Orbital Eccentricity Correction

The viewing angle can be expressed in terms of the tangent point and spacecraft radius as

$$\cos\beta = \frac{R_T}{R_s} \tag{6}$$

Where R_T is the sum of the tangent point altitude and the earth radius. If the spacecraft radius is changed, the viewing angle must change to maintain R_T constant:

$$\Delta\beta = \frac{d\beta}{dR_s}\Delta R_s \tag{7}$$

The derivative is evaluated using equation 6

$$\begin{aligned} -\sin\beta\, d\beta &= -\frac{R_t}{R_s^2}dR_s \\ \frac{d\beta}{dR_s} &= \frac{R_t}{R_s^2\sin\beta} \end{aligned} \tag{8}$$

R_T is replaced using equation 6, resulting in the final expression for the derivative, evaluated at the nominal spacecraft radius:

$$\frac{d\theta}{dR_s} = \frac{1}{R_{s0}\tan\theta} \tag{9}$$

The correction for orbital eccentricity is performed by determining the spacecraft radius R_s by summing the spacecraft altitude and the earth radius, Table 1, (p. 5) for the spacecraft latitude. Both the spacecraft altitude and the spacecraft geodetic latitude are included in the spacecraft status message. Having these quantities and the nominal viewing angle, θ_0, the correction to the viewing angle for spacecraft orbital eccentricity is

$$\begin{aligned}\Delta\theta &= \frac{1}{R_{s_0}}\left(\frac{180}{\pi\tan\theta_0}\right)\left(R_s - R_{s_0}\right) \\ &= E_\theta\left(R_s - R_{s_0}\right)\end{aligned} \tag{10}$$

The value of the correction factor E_θ is given for a range of viewing angles in Table 2, for a nominal spacecraft altitude of 625 km. Equation 10 can be used to determine the compensation factor for other nominal altitudes if the orbit is changed. Using the values found in Table 2, the adjustment to a nominal viewing angle of 23.0 degrees is 0.4819 degrees when the spacecraft is 25 km high and -0.1927 when the spacecraft is 10 km low.

Table 2, Spacecraft Altitude Compensation Factor

viewing angle, degrees	Compensation factor
13	0.03544
14	0.03281
15	0.03053
16	0.02853
17	0.02676
18	0.02518
19	0.02376
20	0.02248
21	0.02131
22	0.02025
23	0.01927
24	0.01838
25	0.01755
26	0.01677
27	0.01606
28	0.01539
29	0.01476

Table 2, Spacecraft Altitude Compensation Factor

viewing angle, degrees	Compensation factor
30	0.01417
31	0.01362
32	0.01309
33	0.01260

6. Oblateness Correction

When oblateness is considered, the geometry becomes a little more involved. Figure 2, shows the effect of earth oblateness on the viewing geometry. In this figure the angle θ is the viewing angle that results in the desired tangent altitude Z_t for the spherical earth case. The true radius of the earth at the tangent point is less than the equatorial radius by ΔR.

$$\Delta R = R_e - R(\phi_t) \tag{11}$$

The distance between the satellite and the nominal tangent point, L, depends on the viewing direction θ and the spacecraft radius R_s.

$$L = R_s \sin\theta \tag{12}$$

Having this length, the correction to the viewing angle is

$$\Delta\theta = \arctan\left(\frac{R_e - R(\phi_t)}{R_s \sin\theta}\right) \tag{13}$$

Once the latitude of the tangent point is developed as a function of the spacecraft latitude, this expression can be used to construct the tables required for oblateness compensation.

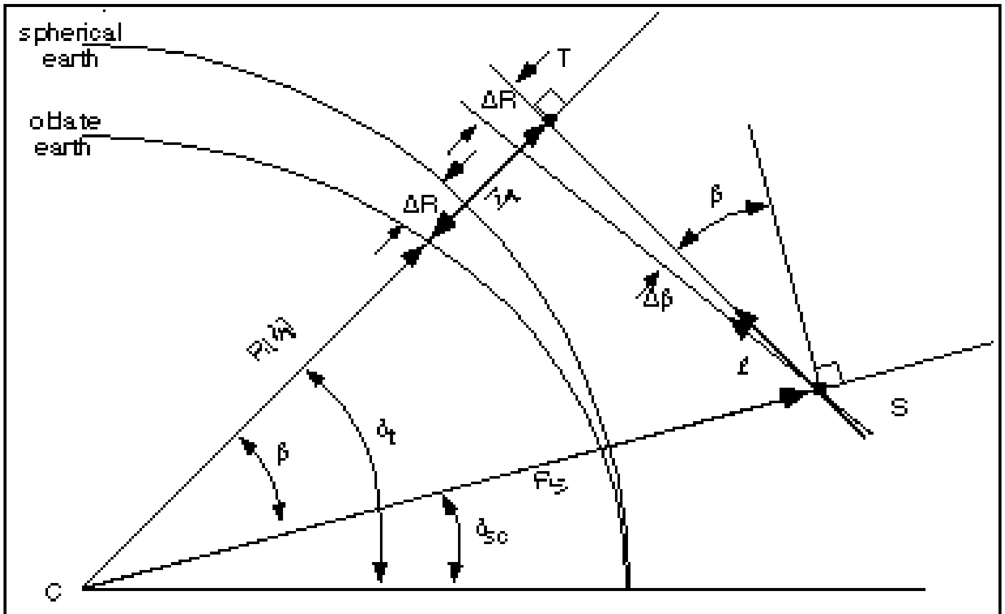

Figure 2, Oblateness Effect

6.1 Tangent Point Location

The ground track of the satellite is shown schematically in Figure 3. In the figure point P is the orbit perigee and points 1 and 2 are satellite positions on the ascending and descending legs of the orbit. The angle along the orbit track from the equator to the perigee is the argument of perigee □. The angle from perigee to the satellite position is the true anomaly v. The orbit plane is inclined to the equator by the inclination angle i. The longitude of the spacecraft is □, and the longitude increment from the ascending node to the satellite position is □□□, where □ is the longitude of the ascending node.

Spherical trigonometric relations can be used to obtain the latitude □ and the longitude □ of the spacecraft in terms of the orbital inclination and the position in the orbit.

$$\sin\square = \sin(\square + v)\sin i$$
$$\tan(\square\ \square\square) = \cos i \tan(\square + v) \tag{14}$$

Defining the track angle t as □+v, the latitude and longitude are

$$\square = \arcsin\left(\sin t \sin i\right)$$
$$\square = \arctan\left(\cos i \tan t\right) + \square \tag{15}$$

In order to determine the increment in latitude and longitude between the spacecraft and the tangent point, the direction of the line of sight must be known with respect to the earth. The telescope azimuth $\square_0$ is, neglecting attitude errors, the angle between the spacecraft velocity vector and the line of sight. The geographic azimuth angle A, the angle between the line of sight and an eastward pointing vector, is

$$A = \square\square\square_0 \tag{16}$$

where the heading angle $\square$ is the angle between the spacecraft velocity and an eastward pointing vector. The tangent of $\square$ is obtained as follows:

$$\begin{aligned} \tan\square &= \frac{d\square}{d\square} \\ &= \frac{d\square/dt}{d\square/dt} \end{aligned} \tag{17}$$

The derivatives of the latitude and longitude with respect to the track angle are

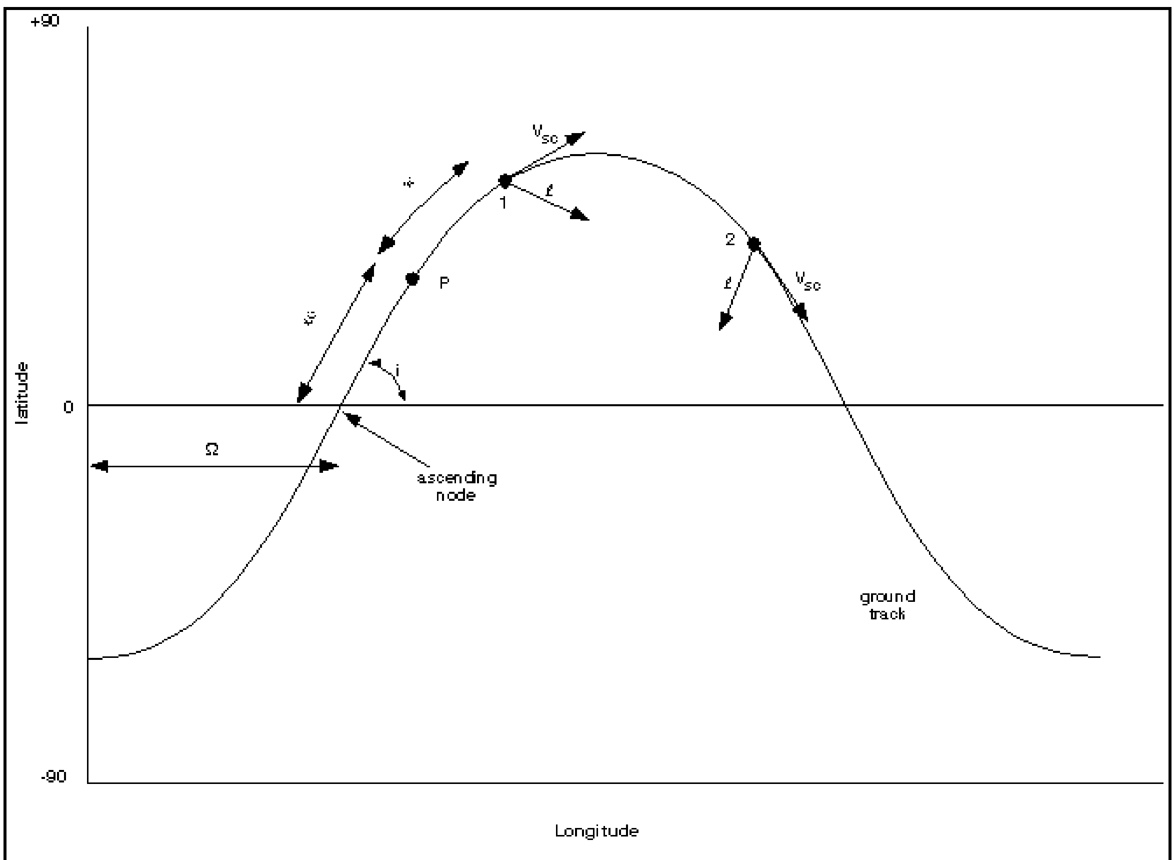

Figure 3, Ground Track Geometry

$$\frac{d\square}{dt} = \sqrt{\frac{\sin^2 i \cos^2 t}{1 - \sin^2 i \sin^2 t}}$$
$$\frac{d\square}{dt} = \frac{\cos i}{\cos^2 t + \cos^2 i \sin^2 t} \tag{18}$$

The tangent of the spacecraft heading vector is obtained

$$\tan\square = \frac{\cos^2 t + \cos^2 i \sin^2 t}{\cos i}\sqrt{\frac{\sin^2 i \cos^2 t}{1 - \sin^2 i \sin^2 t}} \tag{19}$$

which can be reduced, using trigonometric identities to

$$\tan\square = \sqrt{\tan^2 i \cos^2 t\left(1 - \sin^2 i \sin^2 t\right)}$$

The expression for the latitude $\square$ in terms of the track angle t and the inclination i, can be used to eliminate the track angle from the expression

$$\sin\square = \sin t \sin i$$
$$\sin t = \sin\square / \sin i \tag{20}$$
$$\cos^2 t = 1 - \sin^2 t = 1 - \frac{\sin^2\square}{\sin^2 i}$$

substituting these expressions and simplifying results in the following expression for the spacecraft heading angle as a function of spacecraft latitude and orbital inclination.

$$\tan\square = \pm\frac{\cos\square}{\cos i}\sqrt{\sin^2 i - \sin^2\square} \tag{21}$$

where the positive root is used for the ascending leg of the orbit and the negative root for the descending leg. Note that when the spacecraft crosses the equator ($\square$=0), the heading angle is ±i.

The final step is to use spherical trigonometric relations to obtain the latitude and longitude increments between the spacecraft and the tangent point. The spherical triangle is shown in Figure 4. The spacecraft is at point S and the tangent point is at point T. The remaining vertex is a right angle. The hypotenuse has length $\square$ and makes angle A with the side connecting the right angle and the spacecraft.

The sides of this triangle are the increments in latitude and longitude between the spacecraft and the tangent point:

$$\sin\square\square = \sin\square \sin A$$
$$\tan\square\square = \cos A \tan\square \tag{22}$$

Revision D Note: This derivation is only valid near the equator, away from the equator, the triangle is not a right triangle. Appendix II provides the general derivation. The use of equation 22 in the flight software results in miss-locating the tangent point slightly and an error in the Earth radius at the tangent point. The radius error is less than 1 km except at high latitudes. Since the actual pointing information is in the telemetry, this error does not affect the a posteriori knowledge of tangent point location.

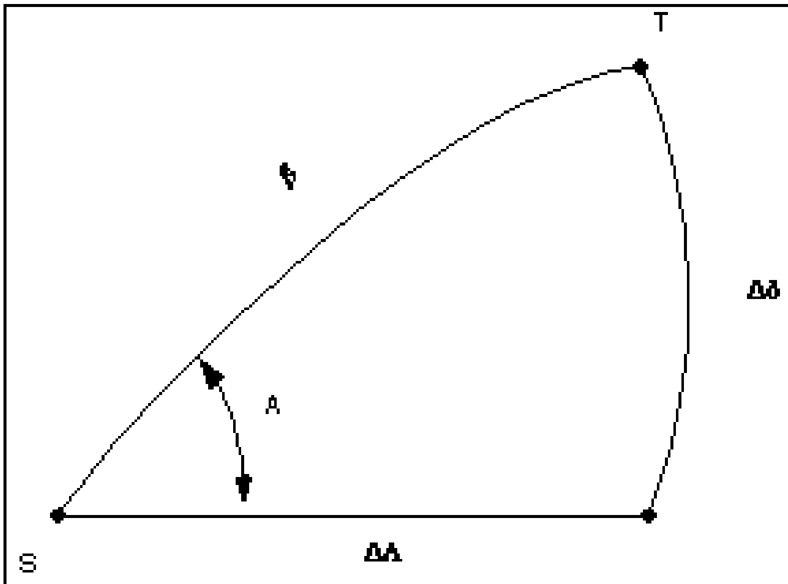

Figure 4, Latitude and Longitude Intervals

6.2 Construction of Compensation Tables

A compensation table for each telescope can be constructed using the equations derived above in the following manner. First, recalling that the spacecraft latitude never exceeds the orbital inclination, the location of the tangent point for each spacecraft latitude in the table is computed using equation 16 to determine the increment of latitude to be applied to each spacecraft latitude. Equations 5 and 4 are used to obtain the radius of the earth at each tangent point latitude. The final step is to calculate the oblateness corrections using equation 8. This process must be performed for each telescope azimuth and for both the ascending and descending legs of the orbit.

A collection of IDL procedures has been developed to construct these tables. Appendix I contains 4 sample tables, computed for every 5 degrees of spacecraft latitude. The tables are applicable to different telescope depending on whether the spacecraft is on the ascending or descending leg of the orbit an whether the spacecraft is in forward or reverse flight. The conversion from elevation angle to encoder steps is 0.004884 degrees per encoder step. Forward flight is when the spacecraft X axis and the spacecraft velocity are in the same direction, and reverse flight is when they are in opposite directions. The ascending leg is when the spacecraft velocity has a northward component and the descending leg the spacecraft velocity has a southward component.

Table 3, Oblateness Compensation Table Selection

	Forward Flight		Backward Flight	
orbit leg	ascending	descending	ascending	descending
telescope 1 (A300)	table A	table B	table C	table D
2 (A301)	table B	table A	table D	table C
3 (A302)	table C	table D	table A	table B
4 (A303)	table D	table C	table B	table A

7. Attitude Errors

7.1 Telescope Elevation Angle

The relationship between the telescope gimbal elevation $\square$ and viewing direction $\square$ is

$$\sin\square = \sin\square\square\square\cos\square_0\cos\square + \square\sin\square_0\cos\square \qquad (5)$$

the viewing angle measured from the local horizontal is affected by the pitch and roll attitude of the spacecraft. In this section, the compensation $\square\square$ required to correct for the actual spacecraft attitude is derived.

Let the desired telescope gimbal angle be

$$\square + \square\square = \square \qquad (23)$$

then substituting into equation (3) yields

$$\begin{aligned}\sin\square &= \sin(\square\square\square\square)\square(\square\cos\square_0\square\square\sin\square_0)\cos\square\\ &= \sin\square\cos\square\square\square\square\cos\square\sin\square\square\square\square(\square\cos\square_0\square\square\sin\square_0)\cos\square\end{aligned} \qquad (24)$$

since the attitude errors are small we can assume that $\square$ and $\square$ are nearly equal so $\square\square$ is small and

$$\begin{aligned}\cos\square\square\square 1\\ \sin\square\square\square\square\square\\ \cos\square\square\cos\square\end{aligned} \qquad (25)$$

making the indicated substitutions,

$$\begin{aligned}\sin\square &= \sin\square\square\square\square\cos\square\square(\square\cos\square_0\square\square\sin\square_0)\cos\square\\ \square\square\cos\square &= \square(\square\cos\square_0\square\square\sin\square_0)\cos\square\\ \square\square &= \square\square\cos\square_0 + \square\sin\square_0\end{aligned} \qquad (26)$$

7.2 Construction of Compensation Tables

The attitude compensation tables are constructed using equation 26 and noting that the correction for pitch □ and roll □ are additive. The roll correction is

$$\Box\Box_{\Box} = +\Box\sin\Box_0 \tag{27}$$

and the pitch correction is

$$\Box\Box_{\Box} = \Box\Box\cos\Box_0 \tag{28}$$

where $\Box_0$ is the nominal telescope azimuth. Since the telescope azimuths $\Box_0$ are 45, 135, 225, and 315, the magnitude of the sin $\Box_0$ and cos $\Box_0$ are 2/2, the tables only differ by a sign. With this simplification the correction becomes

$$\Box\Box = \frac{\sqrt{2}}{2}K\Box \tag{29}$$

where □ is the roll or pitch attitude error in radians and K is either +1 or -1. The value of K selected depends on the telescope to which the compensation is applied and the attitude error being corrected. The appropriate values of K are listed in Table 4

Table 4, Value of K to Use in Attitude Compensation		
telescope	roll error	pitch error
1 (A300)	-1	+1
2 (A301)	-1	-1
3 (A302)	+1	-1
4 (A303)	+1	+1

Appendix I Oblateness Compensation Tables

This appendix contains four compensation tables. In the table the column labeled *scLat* is the spacecraft latitude for which the compensation applies. The column labeled *tpLat* is the latitude of the associated tangent point. Both *scLat* and *tpLat* are in degrees. The column labeled *tpRad* is the radius of the oblate earth at the tangent point. The column labeled *dElv* is the elevation correction in degrees and the column *steps* is the correction in 0.005 degree increments.

Also included in this appendix are plots of the four compensation tables. These plots show the elevation angle correction as a function of the spacecraft latitude.

Table 5, Oblateness Compensation Table A

```
Elevation Angle Corrections
Orbital Inclination: 74.10 ascending
Viewing Azimuth: 45.00
Viewing Elevation: 23.00
```

scLat	tpLat	tpRad	dElv	steps
-74.10	-89.86	6356.75	0.4478	92
-74.00	-89.53	6356.75	0.4478	92
-69.00	-79.63	6357.45	0.4332	89
-64.00	-70.29	6359.20	0.3965	81
-59.00	-61.21	6361.75	0.3433	70
-54.00	-52.78	6364.62	0.2831	58
-49.00	-45.11	6367.45	0.2239	46
-44.00	-38.12	6370.03	0.1698	35
-39.00	-31.64	6372.29	0.1225	25
-34.00	-25.55	6374.19	0.0828	17
-29.00	-19.73	6375.72	0.0507	10
-24.00	-14.13	6376.87	0.0265	5
-19.00	-8.70	6377.65	0.0102	2
-14.00	-3.39	6378.06	0.0016	0
-9.00	1.82	6378.12	0.0005	0
-4.00	6.93	6377.83	0.0065	1
1.00	11.95	6377.23	0.0191	4
6.00	16.89	6376.35	0.0376	8
11.00	21.75	6375.22	0.0611	13
16.00	26.50	6373.91	0.0886	18
21.00	31.15	6372.45	0.1191	24
26.00	35.65	6370.91	0.1513	31
31.00	39.97	6369.36	0.1839	38
36.00	44.06	6367.84	0.2156	44
41.00	47.82	6366.44	0.2450	50
46.00	51.15	6365.21	0.2708	55
51.00	53.91	6364.21	0.2916	60
56.00	55.94	6363.50	0.3066	63
61.00	57.22	6363.06	0.3158	65
66.00	57.97	6362.80	0.3211	66
71.00	58.65	6362.58	0.3259	67
74.10	58.06	6362.77	0.3218	66

Table 6, Oblateness Compensation Table B

```
Elevation Angle Corrections
Orbital Inclination: 74.10 ascending
Viewing Azimuth: 135.00
Viewing Elevation: 23.00
```

scLat	tpLat	tpRad	dElv	steps
-74.10	-89.86	6356.75	0.4478	92
-74.00	-89.47	6356.75	0.4478	92
-69.00	-89.15	6356.76	0.4477	92
-64.00	-86.03	6356.86	0.4456	91
-59.00	-81.88	6357.18	0.4388	90
-54.00	-76.96	6357.85	0.4249	87
-49.00	-71.63	6358.89	0.4030	83
-44.00	-66.15	6360.27	0.3741	77
-39.00	-60.66	6361.92	0.3396	70
-34.00	-55.22	6363.75	0.3013	62
-29.00	-49.86	6365.69	0.2608	53
-24.00	-44.56	6367.65	0.2196	45
-19.00	-39.33	6369.59	0.1790	37
-14.00	-34.16	6371.43	0.1404	29
-9.00	-29.04	6373.13	0.1049	21
-4.00	-23.98	6374.63	0.0735	15
1.00	-18.96	6375.90	0.0470	10
6.00	-14.00	6376.90	0.0261	5
11.00	-9.08	6377.61	0.0111	2
16.00	-4.22	6378.02	0.0025	1
21.00	0.59	6378.13	0.0001	0
26.00	5.33	6377.95	0.0039	1
31.00	10.01	6377.50	0.0135	3
36.00	14.61	6376.79	0.0283	6
41.00	19.15	6375.85	0.0479	10
46.00	23.65	6374.72	0.0716	15
51.00	28.21	6373.39	0.0994	20
56.00	33.00	6371.83	0.1321	27
61.00	38.35	6369.95	0.1715	35
66.00	44.60	6367.64	0.2199	45
71.00	51.91	6364.93	0.2766	57
74.10	58.06	6362.77	0.3218	66

Table 7, Oblateness Compensation Table C

```
Elevation Angle Corrections
Orbital Inclination: 74.10 ascending
Viewing Azimuth: 225.00
Elevation: 23.00
```

scLat	tpLat	tpRad	dElv	steps
-74.10	-58.06	6362.77	0.3218	66
-74.00	-58.47	6362.64	0.3246	66
-69.00	-58.37	6362.67	0.3239	66
-64.00	-57.71	6362.89	0.3193	65
-59.00	-56.80	6363.20	0.3127	64
-54.00	-55.22	6363.75	0.3013	62
-49.00	-52.89	6364.58	0.2839	58
-44.00	-49.88	6365.67	0.2610	53
-39.00	-46.36	6366.98	0.2336	48
-34.00	-42.45	6368.44	0.2031	42
-29.00	-38.27	6369.98	0.1709	35
-24.00	-33.87	6371.53	0.1383	28
-19.00	-29.30	6373.05	0.1066	22
-14.00	-24.61	6374.45	0.0772	16
-9.00	-19.82	6375.70	0.0512	10
-4.00	-14.93	6376.73	0.0296	6
1.00	-9.95	6377.50	0.0133	3
6.00	-4.89	6377.98	0.0033	1
11.00	0.25	6378.14	0.0001	0
16.00	5.50	6377.94	0.0041	1
21.00	10.85	6377.39	0.0158	3
26.00	16.35	6376.46	0.0353	7
31.00	22.03	6375.15	0.0626	13
36.00	27.94	6373.47	0.0977	20
41.00	34.18	6371.43	0.1406	29
46.00	40.85	6369.03	0.1907	39
51.00	48.09	6366.34	0.2471	51
56.00	56.06	6363.46	0.3074	63
61.00	64.78	6360.66	0.3660	75
66.00	74.03	6358.38	0.4137	85
71.00	83.35	6357.04	0.4418	90
74.10	89.86	6356.75	0.4478	92

Table 8, Oblateness Compensation Table D

```
Elevation Angle Corrections
Orbital Inclinations: 74.10 ascending
Viewing Azimuth: 315.00
Elevation: 23.00
```

scLat	tpLat	tpRad	dElv	steps
-74.10	-58.06	6362.77	0.3218	66
-74.00	-57.47	6362.97	0.3175	65
-69.00	-48.85	6366.05	0.2531	52
-64.00	-41.97	6368.62	0.1994	41
-59.00	-36.12	6370.75	0.1548	32
-54.00	-31.04	6372.49	0.1184	24
-49.00	-26.37	6373.95	0.0878	18
-44.00	-21.85	6375.20	0.0616	13
-39.00	-17.34	6376.25	0.0395	8
-34.00	-12.78	6377.10	0.0218	4
-29.00	-8.14	6377.71	0.0090	2
-24.00	-3.44	6378.06	0.0017	0
-19.00	1.33	6378.13	0.0003	0
-14.00	6.16	6377.89	0.0052	1
-9.00	11.04	6377.36	0.0164	3
-4.00	15.98	6376.53	0.0337	7
1.00	20.96	6375.42	0.0570	12
6.00	26.00	6374.06	0.0855	18
11.00	31.08	6372.47	0.1187	24
16.00	36.22	6370.71	0.1555	32
21.00	41.41	6368.83	0.1950	40
26.00	46.67	6366.87	0.2360	48
31.00	51.99	6364.90	0.2772	57
36.00	57.39	6363.00	0.3170	65
41.00	62.85	6361.23	0.3540	72
46.00	68.35	6359.68	0.3864	79
51.00	73.79	6358.43	0.4127	84
56.00	79.00	6357.54	0.4314	88
61.00	83.65	6357.02	0.4423	91
66.00	87.40	6356.80	0.4469	91
71.00	89.91	6356.75	0.4478	92
74.10	89.86	6356.75	0.4478	92

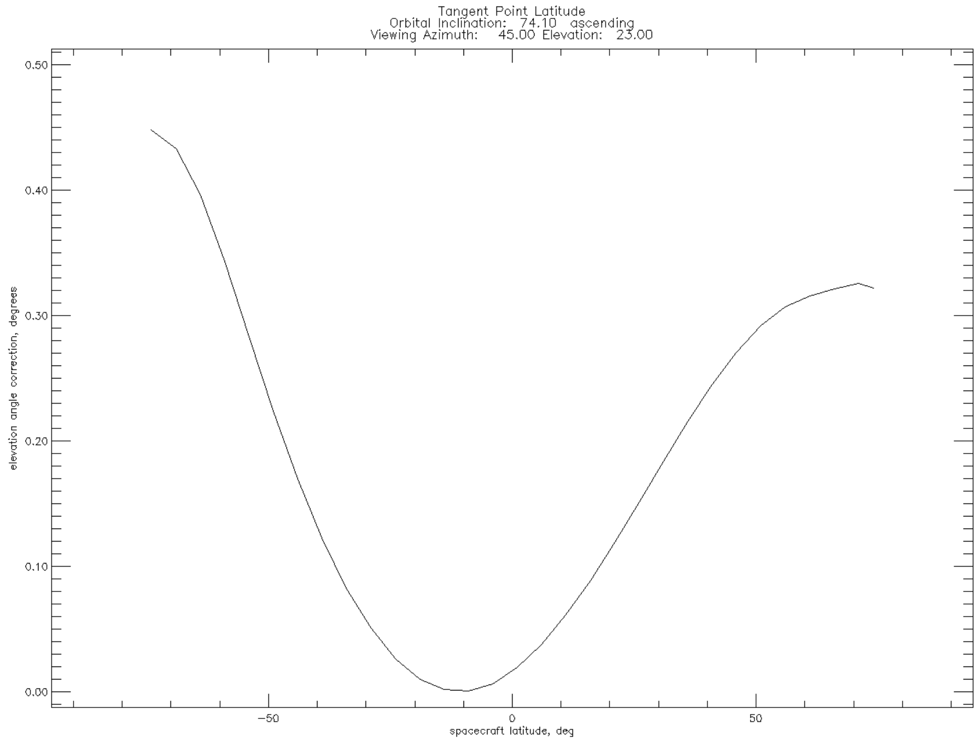

Figure 5, Oblateness Compensation Table A

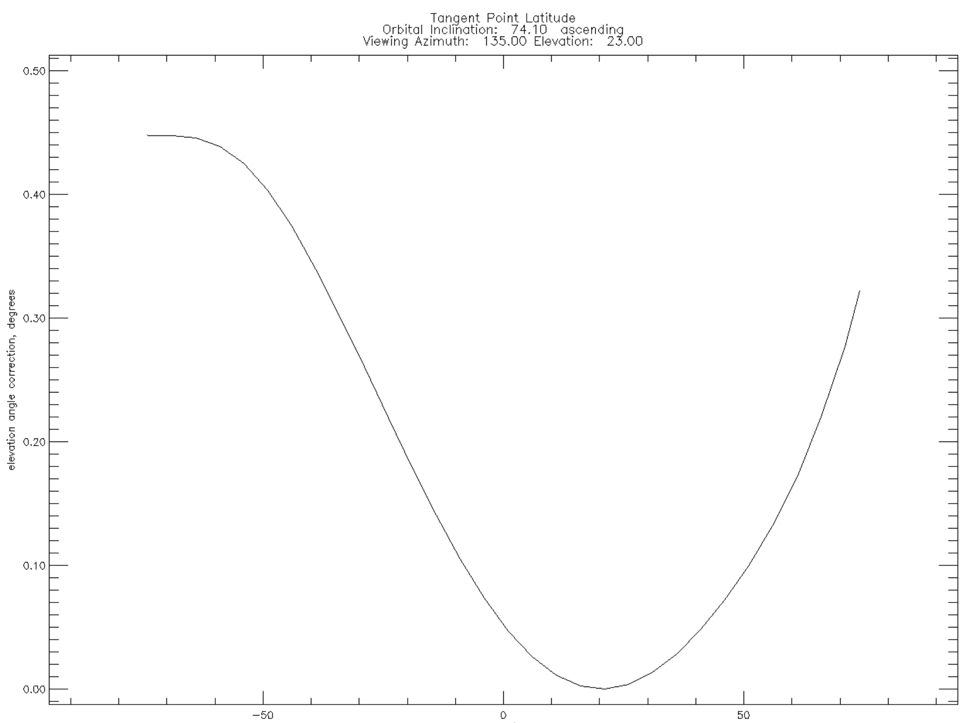

Figure 6, Oblateness Compensation Table B

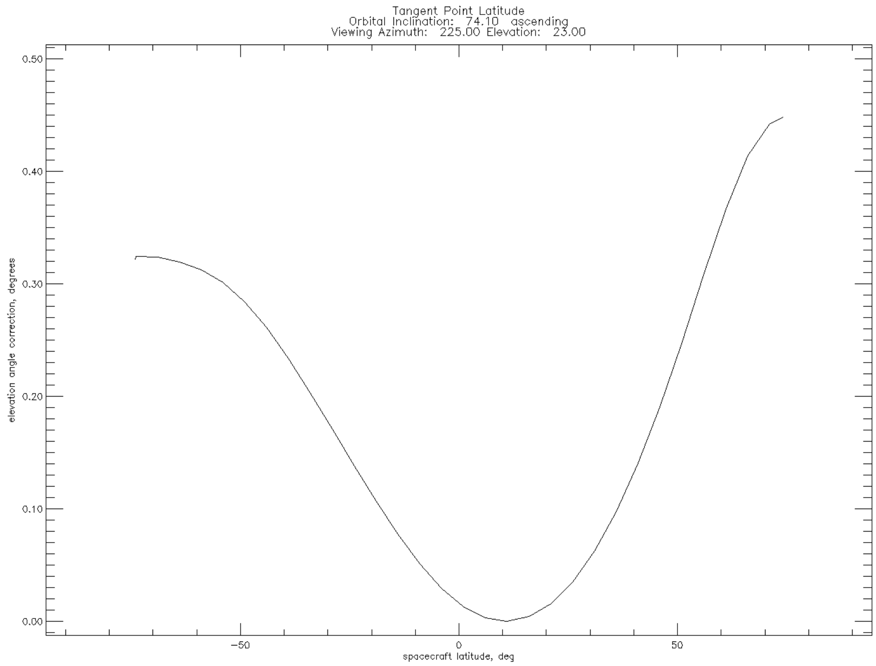

Figure 7, Oblateness Compensation Table C

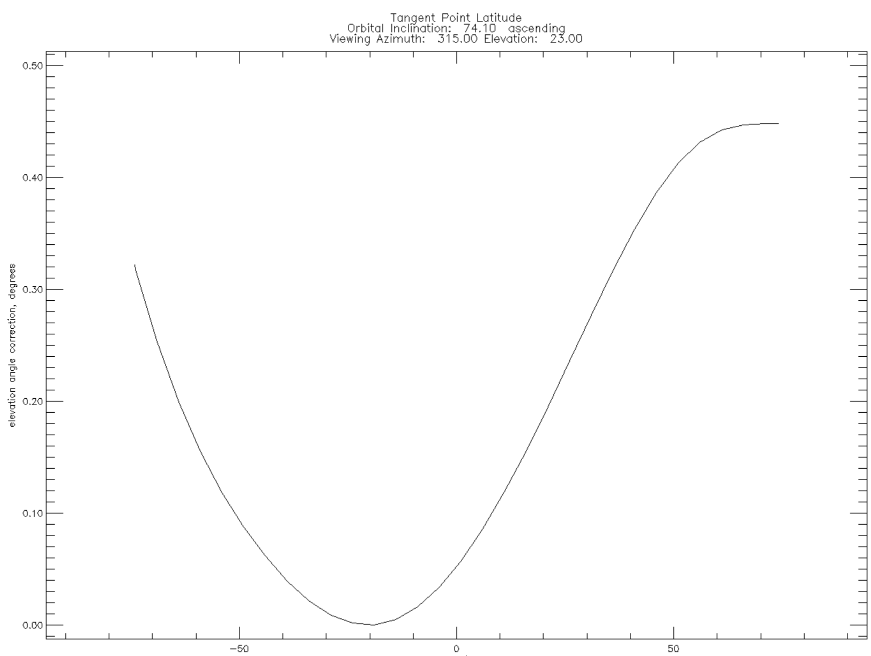

Figure 8, Oblateness Compensation Table D

Appendix II Tangent Point Location

The latitude and longitude of the tangent point can be determined from the spacecraft latitude and longitude, the telescope viewing angle β and the line of sight azimuth A. The spherical triangle is shown in Figure 9, below. In the figure S is the spacecraft location, T is the tangent point location, and P is the Earth's pole. The arcs SP and TP are segments of meridians.

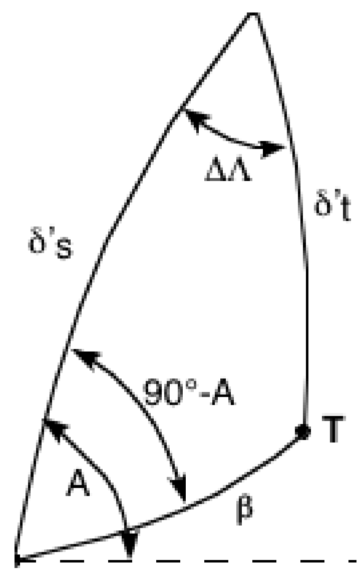

Figure 9, Tangent Point Location

The co-latitude of the tangent point, δ'_t may be determined by the law of cosines for a spherical triangle:

$$\cos\delta'_t = \cos\beta\cos\delta'_s + \sin\beta\sin\delta'_s\cos(90 - A) \tag{30}$$

noting that $\cos\delta' = \sin\delta$ and $\sin\delta' = \cos\delta$, the latitude of the tangent point is

$$\sin\delta_t = \cos\beta\sin\delta_s + \sin\beta\cos\delta_s\sin A \tag{31}$$

The increment in longitude $\Delta\Lambda$ may also be found using the law of cosines

$$\cos\beta = \cos\delta'_s\cos\delta'_t + \sin\delta'_s\sin\delta'_t\cos\Delta\Lambda \tag{32}$$

which may be solved for $\Delta\Lambda$

$$\cos\Delta\Lambda = \frac{\cos\beta - \sin\delta_s\sin\delta_t}{\cos\delta_s\sin\delta_t} \tag{33}$$

To determine the error in latitude, due to the use of equation 22, the tangent point latitude may be written as the sum of the spacecraft latitude and a latitude increment,

If we let $\delta_T = \delta_S + \Delta\delta$, equation 31 can be rewritten as

$$\sin(\lambda_s + \Delta\lambda) = \cos\theta \sin\lambda_s + \sin\theta \cos\lambda_s \sin A \tag{34}$$

Applying trigonometric identities,

$$\sin\lambda_s \cos\Delta\lambda + \cos\lambda_s \sin\Delta\lambda = \cos\theta \sin\lambda_s + \sin\theta \cos\lambda_s \sin A \tag{35}$$

with the spacecraft at the equator, $\lambda S=0$, and equation 34 reduces to equation 22. indicating that equation used in the flight code is accurate at the equator.

Away from the equator, the error in the latitude as calculated increases. The effect of this error in the flight code is to incorrectly determine the latitude of the tangent point, and hence the earth's radius at the tangent point. With an incorrect tangent point radius, the oblateness compensation is incorrect. To evaluate the magnitude of this effect, the tangent point latitudes as determined by equation 22 and by equation 31 were calculated for a series of spacecraft latitudes. The Earth radius at those latitudes were computed and the difference between the radius at the latitude given by equation 22 and the true tangent point latitude given by equation 31. The results are shown in Table 9, Effect of Latitude Calculation Error, below. The values were calculated for a telescope azimuth in the spacecraft frame α_0 of 45° and a typical telescope viewing angle θ of 18°. For convenience an orbital inclination of 75° (rather than the actual TIMED inclination of 74.1°) was used in the computation.

The error in latitude and radius due to using equation 22 rather that the true latitude value increases from zero at the equator to a maximum of about 10° near the pole. The earth radius error increases from zero at the equator to a maximum at high latitude, decreasing slightly when the spacecraft achieves its maximum latitude. The error in tangent point radius is less than 2 km throughout the orbit, and less than 1 km except at high latitudes.

The effect of this error is to misplace the scan range by one altitude step (of 2.5 km) near the poles. Since the actual elevation angles are included in the telemetry, this error does not affect the a posteriori tangent point location knowledge.

Table 9, Effect of Latitude Calculation Error

$\square_s$	$\square$	A	equation 22		true values		latitude error	radius error
			$\square_s$	R_e	$\square_s$	R_e		
-75.00	0.00	-45.00	-87.62	6356.79	-77.21	6357.81	10.407	1.019
-67.50	22.63	-22.37	-74.26	6358.34	-67.47	6359.92	6.786	1.576
-60.00	39.57	-5.43	-61.68	6361.60	-56.96	6363.15	4.719	1.550
-52.50	52.35	7.35	-50.24	6365.54	-46.93	6366.77	3.310	1.226
-45.00	60.92	15.92	-40.14	6369.29	-37.78	6370.15	2.363	0.860
-37.50	66.49	21.49	-31.00	6372.50	-29.28	6373.05	1.715	0.553
-30.00	70.12	25.12	-22.46	6375.04	-21.22	6375.36	1.244	0.319
-22.50	72.47	27.47	-14.30	6376.84	-13.43	6376.99	0.875	0.150
-15.00	73.94	28.94	-6.40	6377.87	-5.84	6377.92	0.562	0.044
-7.50	74.74	29.74	1.32	6378.13	1.60	6378.12	0.278	-0.005
0.00	75.00	30.00	8.89	6377.63	8.89	6377.63	0.000	0.000
7.50	74.74	29.74	16.32	6376.46	16.03	6376.52	-0.289	0.057
15.00	73.94	28.94	23.60	6374.73	22.99	6374.90	-0.609	0.165
22.50	72.47	27.47	30.70	6372.60	29.71	6372.92	-0.983	0.317
30.00	70.12	25.12	37.54	6370.24	36.10	6370.76	-1.442	0.515
37.50	66.49	21.49	44.00	6367.86	41.97	6368.62	-2.027	0.755
45.00	60.92	15.92	49.86	6365.68	47.09	6366.71	-2.772	1.027
52.50	52.35	7.35	54.76	6363.91	51.13	6365.22	-3.634	1.307
60.00	39.57	-5.43	58.32	6362.69	54.00	6364.18	-4.324	1.496
67.50	22.63	-22.37	60.74	6361.89	56.48	6363.31	-4.270	1.421
75.00	0.00	-45.00	62.38	6361.38	59.55	6362.28	-2.826	0.899