XXII... Auroral Battery Electrical Circuit

Introduction:

Scientists have proposed that changes in the magnetic field in the magnetotail region cause releases of energy that eventually supply the 'battery' to light-up the aurora on Earth.

Let's explore this idea in more detail to sdee what they are talking about!

Part 1: How big a battery do you need?

Energy is stored in a magnetic field, and the amount depends on how strong the field is, and how big a volume it occupies. What we are going to do is estimate just how much magnetic energy is available in the magnetic tail region of Earth.

1) Let's suppose the volume of the magnetotail region is a cylinder with a height of 300,000 kilometers, and a radius of 120,000 kilometers. Use the formula for a cylinder to estimate the magnetotail volume, in cubic meters.

$$
\begin{aligned}
V &= \pi r^2 \ h \\
&= 3.14 \times (1.2\times10^8 \text{meters})^2 \ \times (3\times10^8 \text{meters}) \\
&= 1.3 \times 10^{25} \text{ cubic meters}
\end{aligned}
$$

2) The formula for the energy of a magnetic field is given by the formula to the right in which the strength of the magnetic field, B, is expressed in Teslas, V in cubic meters, and the energy will then be in units of Joules.

$$E = \frac{10^7}{8\pi} B^2 \times V$$

3) For a magnetic field with a strength of 2×10^{-8} Teslas [the typical strength of the magnetotail field] and the volume of space you just calculated in Part 1, the total energy of the magnetotail field is:

$$
\begin{aligned}
E &= 4.0 \times10^5 \times (2 \times 10^{-8} \text{ Teslas})^2 \times 1.3 \times 10^{25} \text{m}^3 \\
E &= 2.1 \times 10^{15} \text{ Joules}
\end{aligned}
$$

Part 2: How much energy do you need to light-up the auroras in the Northern and Southern Hemispheres?

Auroras are powered by currents of electrons with currents of about 1,000,000 Amperes. Your home uses about 200 Amperes at 110 Volts. The atmosphere that this auroral current has to flow through has a resistance of about 0.1 Ohms.

Electrical power is calculated using a formula that relates resistance (R) and current (I) to the power (P) that they can produce in a circuit:

$$\mathbf{P = I^2 \times R \quad Watts}$$

where R is measured in Ohms, and I is in Amperes. From the information given and the power formula, you can calculate the power dissipated by an aurora:

$$\mathbf{P = (\,10^6 \text{ Amperes})^2 \times 0.1 \text{ Ohms}}$$

$$\mathbf{P = 10^{11} \text{ Joules/second}}$$

Part 3: How many seconds can the magnetotail 'battery' continue to supply energy to the aurora to keep them going?

The answer to the first question in Part 1 tells us how much energy is available. The answer to the second question tells us at what rate (energy per second) the aurora are converting energy into light and heat.

1) To find out how long this can continue, divide the answer from question one, by the answer from question two:

$$\mathbf{\text{Time} = \frac{2.1 \times 10^{15} \text{ Joules}}{10^{11} \text{ Joules/second}}}$$

$$\mathbf{= 21{,}000 \text{ seconds, or about } 5.8 \text{ hours}}$$

Although this is only an estimate, it comes very close to the most intense phase of an auroral display which often lasts an entire evening. There have been many approximations used in this “back of the envelope” calculation. Have your students identify them and consider other values. Which ones might be the most uncertain? (Example: dimensions of magnetotail region involved in the conversion of magnetic energy to particle energy, or the amperage of the auroral current.)